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THE PROBLEM OF OBTAINING PRESCRIBED DISTRIBUTIONS OF GAS PARAMETERS†

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Using one-dimensional cylindrically and spherically symmetric flows as examples, the following problem is investigated in which a prescribed density distribution in a gas is obtained: given a known gas flow (background flow), it is required to continuously attach another, unknown gas flow whose density distribution at a fixed instant of time is described by some previously given function. It is shown that this problem is a characteristic Cauchy problem of standard type for which there is a valid analogue of the Kovalevskaya theorem, provided that the input data are analytic. Other problems of the same type are considered: to ensure that the unknown flow will have a prescribed gas velocity distribution and the prescribed density of the gas in the unknown flow is strictly greater than that of the gas in the background flow (flow with discontinuity). On the assumption that the input data to these problems are analytic, the existence and uniqueness of solutions are proved, in fact—of piecewise analytic solutions. The theorems proved are extended to the case of non-one-dimensional flows.

A meaningful example of the problem of obtaining prescribed distributions of parameters in a gas is that of obtaining a prescribed density distribution; this problem is related to the problem of shockless strong compression of a gas [1] and to that of optimal compression of gas layers [2, 3]. In addition, obtaining a prescribed gas velocity distribution may be related to the process of obtaining a high-velocity gas flow.

It will be shown below that in problems of obtaining prescribed distributions of gas-dynamic parameters continuously adjoined to a background flow (BF), one can arbitrarily prescribe the distribution of only one gas-dynamic parameter. For example, at a certain instant of time one can obtain a prescribed distribution of the gas density, but the distributions of the other gas-dynamic parameters will then be uniquely defined. Alternatively, one can prescribe an arbitrary gas velocity at some instant of time; then the density and entropy at that time are uniquely defined. Alternatively, one can prescribe an arbitrary gas velocity at some instant of time; then the density and entropy at that time are uniquely defined. We emphasize that, apart from this one arbitrarily prescribed distribution, the BF is also an arbitrary initial element of the problem.

A second important aspect of the problem of obtaining prescribed gas parameter distributions is that these problems must be solved in the direction of inversely changing time. This approach arises in gas dynamics when one first constructs an exact solution of the system of gas-dynamic equations (using the symmetry properties of the flow, self-similarity, linearity with respect to part of the variables, or the like), and then, for this solution, chooses a meaningful initial-boundary-value problem. Consideration of the solution thus constructed as time varies in the inverse direction then leads in a unique fashion both to the distributions of the gas parameters at earlier times and to the laws of the external forces acting on the gas that "generate" the given gas flow.

The difference between the approach outlined here and the traditional approach is as follows. Any treatment of the problem in inversely varying time will be "based" on the arbitrarily given distribution of one of the gas-dynamic parameters continuously adjoined to the given BF, rather than on the distributions already "dictated" by the exact solution. As will be shown below, the solution of the system of gas-dynamic equations in that case will be constructed uniquely. Then, varying the prescribed distribution and the BF, one can consider other aspects of the problem, such as the domain of existence of the solution, which is determined, among other things, by the mass of the gas in the constructed flow, or the external energy consumption needed to obtain an *a priori* given gas density, or, finally, the physical possibility of creating the BF to which the solution is adjoined.

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1. OBTAINING A PRESCRIBED DISTRIBUTION OF GAS DENSITY CONTINUOUSLY ADJOINING A BACKGROUND FLOW

First, for clarity and simplicity, we will consider the system of equations

$$\begin{aligned} &\sigma_t + u\sigma_r + \delta\sigma\left(u_r + \nu\frac{u}{r}\right) = 0 \\ &u_t + \frac{1}{\delta}s^2\sigma\sigma_r + uu_r + \frac{2}{\gamma}\sigma^2 ss_r = 0 \\ &s_t + us_r = 0 \\ &\left(\sigma = \rho^\delta > 0,\ \delta = \frac{(\gamma-1)}{2},\ s = A(S) > 0,\ r = (x_1^2 + \ldots + x_{\nu+1}^2)^{1/2}\right) \end{aligned} \tag{1.1}$$

describing one-dimensional cylindrically ($\nu = 1$) and spherically ($\nu = 2$) symmetric flows of an ideal polytropic gas. Here ρ is the gas density, γ is the constant in the equation of state $p = A^2(S)\rho^\gamma/\gamma$, $\gamma > 1$, where p is the pressure S is the entropy and u is the gas velocity. The speed of sound in the gas is given by $c = s\sigma$. Note that the theorems proved below also transfer to the case of a normal gas with an arbitrary equation of state $p = p(S, \rho)$.

We introduce the vector of unknown functions

$$\mathbf{U} = \begin{Vmatrix} s \\ u \\ \sigma \end{Vmatrix}$$

Suppose that in some neighbourhood of a point ($t = t_*, r = r_*$), $r_* > 0$ we have a given background flow (BF)

$$\mathbf{U} = \mathbf{U}^0(t, r)$$

where the components of the vector $\mathbf{U}^0(t, r)$—the functions $s^0(t, r)$, $u^0(t, r)$, $\sigma^0(t, r)$—constitute a solution of system (1.1). The sonic lines $C^\pm$ of the given BF passing through the point ($t = t_*, r = r_*$) are then also known. Choose one of them: $r = r_0(t)$, say C^+. Then the BF is considered to the right of C^+ and the new unknown flow to the left of C^+. Had we chosen C^- as the "separating" sonic line, the BF would have been to the left of C^- and the unknown flow to the right of C^-. The values of the gas-dynamic parameters of the BF $\mathbf{U}^0(t, r)\,|_{r=r_0(t)} = \mathbf{U}_{00}(t)$ are known on the selected sonic line (for which we retain the notation $C^\pm$), that is,

$$s\,|_{C^\pm} = s_{00}(t), \quad u\,|_{C^\pm} = u_{00}(t), \quad \sigma\,|_{C^\pm} = \sigma_{00}(t) \tag{1.2}$$

We now supplement system (1.1) with "initial data" (1.2), and also with one additional "boundary condition"

$$\sigma\,|_{t=t_*} = \sigma_*(r) \tag{1.3}$$

The function $\sigma = \sigma_*(r)$ is related by the formula $\sigma_*(r) = \rho_*^\delta(r)$ to the required gas density distribution $\rho = \rho_*(r)$, which takes a prescribed value at time $t = t_*$. It is assumed that the prescribed density distribution is continuously matched at time $t = t_*$ and at the point $r = r_*$ to the gas density of the BF, that is

$$\sigma_*(r_*) = \sigma_{00}(t_*)$$

Note that the functions indicated in (1.2) satisfy the following differential equation [4]

$$\frac{dr_0(t)}{dt} = u_{00}(t) \pm s_{00}(t)\sigma_{00}(t) \tag{1.4}$$

Theorem 1. Problem (1.1)–(1.3) is a characteristic Cauchy problem (CCP) of standard form; if the functions

$$r_0(t),\quad s_{00}(t),\quad u_{00}(t),\quad \sigma_{00}(t),\quad \sigma_*(r)$$

are analytic, the problem has a unique analytic solution in some neighbourhood of the point ($t = t_*$, $r = r_*$).

In order to reduce the CCP as formulated to standard form [5, 6], we make the change of variables $x = r - r_0(t)$, $y = t - t_*$ with Jacobian equal to unity, on the assumption that $r'_0(t)$ is finite. With this transformation the sonic line $C^{\pm}$ becomes the coordinate axis $x = 0$, and the time $t = t_*$ becomes the axis $y = 0$. As a result, the CCP is transformed as follows:

$$\begin{aligned}
&\tilde{u}\sigma_x + \delta\sigma u_x + \sigma_y + \nu\delta\frac{\sigma u}{[x + r_0(y)]} = 0\\
&\delta s^2\sigma\sigma_x + \tilde{u}u_x + \frac{2}{\gamma}\sigma^2 s s_x + u_y = 0\\
&\tilde{u}s_x + s_y = 0\\
&\sigma(x, y)|_{x=0} = \sigma_{00}(y),\quad u(x, y)|_{x=0} = u_{00}(y)\\
&s(x, y)|_{x=0} = s_{00}(y),\quad \sigma(x, y)|_{y=0} = \sigma_*(x),\quad \sigma_*(0) = \sigma_{00}(0)
\end{aligned}\tag{1.5}$$

where $\tilde{u} = u - r'_0(y)$.

Since condition (1.4) holds on $C^{\pm}$, it follows that

$$\tilde{u}|_{x=0} = \mp s_{00}(y)\sigma_{00}(y) \neq 0$$

in some neighbourhood of the point $y = 0$. Hence the function $\tilde{u}^{-1}$ will be analytic in some neighbourhood of the point ($x = 0, y = 0$). Then, using the third equation of problem (1.5), we can eliminate the derivative s_x from the second equation. Interchanging the equations (for convenience in subsequent operations), we write the system of equations of problem (1.5) in the form

$$A\mathbf{U}_x + B\mathbf{U}_y = \mathbf{C}\tag{1.6}$$

$$\mathbf{U} = \begin{Vmatrix} s \\ u \\ \sigma \end{Vmatrix},\quad A = \begin{Vmatrix} \tilde{u} & 0 & 0 \\ 0 & \tilde{u} & \frac{1}{\delta}s^2\sigma \\ 0 & \delta\sigma & \tilde{u} \end{Vmatrix}$$

$$B = \begin{Vmatrix} 1 & 0 & 0 \\ -\frac{2}{\gamma}\frac{s\sigma^2}{\tilde{u}} & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix},\quad \mathbf{C} = \begin{Vmatrix} 0 \\ 0 \\ -\nu\sigma\frac{\sigma u}{x + r_0(y)} \end{Vmatrix}$$

We now verify that the resulting problem (system (1.6) and the initial-boundary conditions (1.5)) satisfies the conditions of an analogue of Kovalevskaya's theorem [6].

That all the input data of the problem are analytic follows from the form of the system and from the assumption that the functions σ_{00}, u_{00}, s_{00}, σ_*, r_0 are analytic. The 2×2 submatrix in the upper left corner of the matrix A has a non-zero determinant. As matrices $T_1(y)$ and $T_2(y)$, which will now be used to transform system (1.6), we choose the following

$$T_1 = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\delta\sigma_{00}(y) & \tilde{u}_{00}(y) \end{Vmatrix}$$

$$T_2 = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{\delta}s_{00}^2(y)\sigma_{00}(y) \\ 0 & 0 & \tilde{u}_{00}(y) \end{Vmatrix}$$

$$\det T_1|_{y=0} = \det T_2|_{y=0} = \tilde{u}_{00}(y)|_{y=0} \neq 0,\quad \tilde{u}_{00}(y) = u_{00}(y) - r'_0(y)$$

Then the matrix $(T_1AT_2)|_{x=0}$ has the required form, that is, all the elements bordering the upper left 2×2 minor vanish. The element in the lower right corner of the matrix $(T_1BT_2)|_{x=0}$ is $2s_{00}^2(y)\sigma_{00}^2(y)$, that is, it does not vanish when $y = 0$. The vector of the new unknown functions

$$\mathbf{V} = T_2^{-1}\mathbf{U} = \frac{1}{\tilde{u}_{00}} \cdot \begin{Vmatrix} \tilde{u}_{00} & 0 & 0 \\ 0 & \tilde{u}_{00} & \dfrac{s_{00}^2\sigma_{00}}{\delta} \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} s \\ u \\ \sigma \end{Vmatrix} = \begin{Vmatrix} s \\ u + \dfrac{s_{00}^2\sigma_{00}}{\delta\tilde{u}_{00}}\sigma \\ \dfrac{1}{\tilde{u}_{00}}\sigma \end{Vmatrix} = \begin{Vmatrix} v_1 \\ v_2 \\ v_3 \end{Vmatrix}$$

is such that specification of a condition at $y = 0$ for the function σ is equivalent to specifying a condition at $y = 0$ for the function v_3.

Consequently, the CCP considered for system (1.6), and hence the CCP (1.5) and the CCP (1.1)–(1.3), are CCPs of standard form, for which we have a valid analogue of Kovalevskaya's theorem [6].

Theorem 1 guarantees the existence of an analytic solution of the CCP (1.1)–(1.3) in some complete neighbourhood of the point $(t = t_*, r = r_*)$. But, as already noted above, the solution has to be used only in a "quarter" of that neighbourhood: to one side of the sonic line $C^\pm$ (to the other side of $C^\pm$ is the BF) and for $t \leqslant t_*$ (the aim of the problem is to obtain the required distribution $\sigma_*(r)$ up to the time $t = t_*$).

In the neighbourhood of the point $(t = t_*, r = r_*)$, the solution may be represented uniquely as a double series in powers $(t - t_*)^k(r - r_*)^l$ with constant coefficients. The solution of the CCP (1.1)–(1.3) may also be represented uniquely [6] as a series in powers $[r - r_0(t)]^k$, but in this case the coefficients are functions of t. And, if the BF is a homogeneous gas at rest, then, using the methods of [7, 8], we can prove that the convergence domain of this last series is unbounded as t varies. In some rare cases, the solution of the CCP (1.1)–(1.3) may be expressed in terms of quadratures or even as a finite formula.

We stress once again that Theorem 1 is local in nature and therefore the flow may well have singularities outside the aforementioned neighbourhoods, such as infinite gradients and, consequently, shock waves.

Using the solution of the CCP (1.1)–(1.3), which is unique, we can uniquely reproduce the laws governing the external forces acting on the gas which lead at time $t = t_*$ to the prescribed density distribution $\rho = \rho_*(r)$.

For example, in the domain of existence of the solution of problem (1.1)–(1.3), we choose a point on the sonic line $C^\pm$, say $(t = t_0, r = r_0(t))$, $t_0 < t_*$, and solve the Cauchy problem

$$\frac{dr^0(t)}{dt} = u(t, r^0(t)), \quad r^0(t_0) = r_0(t_0) = r_0^0$$

If the function $u(t, r)$ on the right-hand side of the equation is the gas flow velocity constructed by solving the CCP (1.1)–(1.3), the, first, the problem has a unique analytic solution; and, second, the function $r = r^0(t)$ describes the trajectory of motion of a certain gas particle. This trajectory may be taken as the trajectory of motion of an impermeable piston starting at time $t = t_0$ from the point $r = r^0(t_0)$ and producing the prescribed density distribution up to time $t = t_*$.

Suppose the BF is fixed (for example, a homogeneous gas at rest) and let the solution of the CCP (1.1)–(1.3) be a compression wave with strictly monotone distribution (1.3) of the gas density at time $t = t_*$. It can easily be proved that one can then take the point (t_0, r_0^0) so close to the point (t_*, r_*) that the above trajectory of motion of the gas particle does not leave the domain existence of the solution of the CCP (1.1)–(1.3) for $t_0 \leqslant t \leqslant t_*$. This is because, locally, the solution of the CCP (1.1)–(1.3), and hence also the gas velocity in the solution, are determined by the first terms of the series, whose analysis leads to this assertion.

If the solution of the CCP (1.1)–(1.3) is an expansion wave, the solution of the last differential equation must be constructed by going backwards in time from a point (t_*, r_1^0) lying in the domain of existence of the solution of the CCP (1.1)–(1.3) sufficiently close to the point (t_*, r_*). Then the trajectory of motion of the gas particle will at some time $t = t_0 < t_*$ intersect the sonic line $C^\pm$ and will also not leave the domain of existence of the solution of the CCP (1.1)–(1.3).

Note that in both cases the piston will be in contact at time $t = t_0$ only with the BF, which is an arbitrary element (such as a homogeneous gas at rest) in the problem of obtaining prescribed distributions.

In some physical experiments it is easiest simply to maintain the necessary pressure at a given point of space. To find the required law, it is sufficient to consider a solution of the CCP (1.1)–(1.3) at a fixed point $r = r_0^0$ on the appropriate side of the sonic line $C^{\pm}$.

Note that if the prescribed distribution to be obtained at time $t = t_*$ is not that of the function σ but that of the function u, that is,

$$u(t, r)\,|_{t=t_*} = u_*(r), \quad u_*(r_*) = u_{00}(t_*)$$

then the new CCP

$$\tilde{u}s_x + s_y = 0$$

$$\tilde{u}\sigma_x + \delta\sigma u_x + \sigma_y + \nu\delta\frac{\sigma u}{[x + r_0(y)]} = 0$$

$$\frac{1}{\delta}s^2\sigma\sigma_x + \tilde{u}u_x - \frac{2}{\gamma}\frac{s\sigma^2}{\tilde{u}}s_y + u_y = 0$$

$$s(x, y)\,|_{x=0} = s_{00}(y), \quad \sigma(x, y)\,|_{x=0} = \sigma_{00}(y), \quad u(x, y)\,|_{x=0} = u_{00}(y)$$

$$u(x, y)\,|_{y=0} = u_*(x), \quad u_*(0) = u_{00}(0)$$

will also be a CCP of standard form, for which the analogue of Kovalevskaya's theorem is valid [5, 6]. This may be verified directly by defining new matrices $T_1^1(y)$, $T_2^1(y)$ as follows:

$$T_1^1 = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\delta\sigma_{00} & \tilde{u}_{00} \end{Vmatrix}, \quad T_2^1 = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & -\delta\sigma_{00} \\ 0 & 0 & \tilde{u}_{00} \end{Vmatrix}$$

And, finally, we mention a last fact relating to the CCP (1.1)–(1.3). If we multiply the system of equations (1.6) on the left by T_1 and set $x = 0$, the coefficients of all the derivatives issuing from the surface $x = 0$, namely, s_x, u_x and σ_x, will vanish identically. Therefore, the resulting relationship will be a necessary condition for the CCP to be solvable with initial data on the sonic line for the system of equations of gas dynamics in the cylindrically and spherically symmetric case

$$\frac{\gamma - 1}{\gamma}\sigma_{00}s_{00}' + s_{00}\sigma_{00}' + \nu\delta\frac{s_{00}\sigma_{00}u_{00}}{r_0} \pm \delta u_{00}' = 0 \tag{1.7}$$

and this relationship involves only derivatives of the unknown functions which are interior relative to the surface $x = 0$. In deriving this equality allowance was made for (1.4) and for the fact that in these problems $\sigma_{00}(y) > 0$.

We recall that the functions σ_{00}, s_{00}, u_{00} in Eq. (1.7) actually depend on time ($y = t - t_*$), and they give the values of the gas-dynamic parameters on the sonic line $C^{\pm}$: $r = r_0(y)$. If the functions σ_{00}, s_{00}, u_{00} are the values of some solution of the system of equations of gas dynamics considered on $C^{\pm}$, they automatically satisfy Eq. (1.7). But if they are chosen arbitrarily, then, by condition (1.4) (which guarantees that the curve $r = r_0(t)$ is a sonic line), condition (1.7) must be imposed upon $u|_{t=t^*}$ as an additional differential constraint.

The proof of Theorem 1 (and its analogue when σ_{00}, s_{00} and u_{00} is given) is duplicated almost verbatim in the multi-dimensional case, when the vector of unknown functions $\mathbf{U}$ depends on t, x_1 and x_2 or on t, x_1, x_2 and x_3. When that is done, the transformation to the space of new independent variables (in which the surface $C^{\pm}$ becomes a coordinate plane) is accomplished in a suitable way [5] and it is shown, as before, that the resulting CCP is a CCP of standard form.

2. OBTAINING A PRESCRIBED DISCONTINUOUS GAS DENSITY DISTRIBUTION

Suppose, as before, that the following data are given in some neighbourhood of the point ($t = t_*$, $r = r_*$): a BF $\mathbf{U}^0$, its sonic line $C^{\pm}$ and (1.2)—the values of the parameters of the BF on $C^{\pm}$. Function (1.3) is also given, but it is now assumed that

$$\sigma_*(r_*) > \sigma_{00}(t_*)$$

that is, the prescribed distribution of the density at time $t = t_*$ at the point $r = r_*$ is strictly greater than the density of the BF at the point of attachment.

In order to obtain a discontinuous distribution of the function:

$$\sigma(t, r)|_{t=t_*} = \begin{cases} \sigma_{00}(r) & \text{for } r \text{ in the domain of definition of the BF} \\ \sigma_*(r) & \text{for } r \text{ in the domain of definition of the unknown flow} \end{cases}$$

at the time $t = t_*$, we must attach two new flows to the BF.

The first of the unknown flows is an extension of the concept of a simple centered Riemann wave to the case of cylindrically and spherically symmetric flows. As "initial conditions," this must satisfy Eq. (1.2), and the "boundary condition" at time $t = t_*$ must describe a vertical straight line in the (r, σ) plane

$$r = r_*, \quad \sigma \geqslant \sigma_{00}(t_*) \tag{2.1}$$

The solution of problem (1.1), (1.2), (2.1) also yields a prescribed density distribution continuously attached to the given BF. In this flow, however, as the time $t = t_*$ is approached the gas is compressed without limit: as $t_1 \to t_* - 0$, the graph of the function $\sigma = \sigma(t, r)|_{t=t1}$ will tend to the straight line (2.1).

It is clear that the solution of problem (1.1), (1.2), (2.1) in the space of the variables t and r will have a singularity.

In order to resolve this singularity, we exchange the roles of the dependent and independent variables, taking t and σ as new independent variables and treating r, u and s as new unknown functions. Thus transformed, problem (1.1), (1.2), (2.1) becomes

$$\begin{aligned} & r(u - r_t) + \delta\sigma(ru_\sigma + \nu ur_\sigma) = 0 \\ & r_\sigma u_t + (u - r_t)u_\sigma + \frac{2}{\gamma}\sigma^2 ss_\sigma + \frac{1}{\delta}\sigma s^2 = 0 \\ & r_\sigma s_t + (u - r_t)s_\sigma = 0 \\ & r(t, \sigma)|_{C^\pm} = r_0(t), \quad u(t, \sigma)|_{C^\pm} = u_{00}(t) \\ & s(t, \sigma)|_{C^\pm} = s_{00}(t), \quad r(t, \sigma)|_{t=t_*} = r_*, \quad r_* = \text{const} > 0 \end{aligned} \tag{2.2}$$

where the sonic line $C^\pm$ is given by the equation $\sigma = \sigma_{00}(t)$ and the functions $r_0(t)$, $\sigma_{00}(t)$, $s_{00}(t)$ and $u_{00}(t)$ satisfy conditions (1.4) and (1.7).

Theorem 2. Problem (2.2) is a CCP of standard form which, provided the functions

$$r_0(t), \quad \sigma_{00}(t), \quad u_{00}(t), \quad s_{00}(t)$$

are analytic, has a unique analytic solution in some neighbourhood of the point $(t = t_*, \sigma = \sigma_{00}(t_*))$.

A proof of Theorem 2 may be found in [9], where the solution of problem (2.2) is used to describe a gas escaping into a vacuum.

If the solution of the CCP (2.2) is expressed as series in powers $(t - t_*)^k$ (with coefficients which are functions of σ), it can be proved by the methods of [7–9] that the convergence domains of the series are unbounded as functions of $\sigma(\sigma \geqslant 0)$.

Then, for any finite value of $\sigma^* = \sigma_*(r_*)$, one can consider the problem

$$\begin{aligned} & r_t(t, \sigma_1(t)) + r_\sigma(t, \sigma_1(t))\frac{d\sigma_1(t)}{dt} = u(t, \sigma_1(t)) \pm s(t, \sigma_1(t))\sigma_1(t) \\ & \sigma_1(t)|_{t=t_*} = \sigma^* \end{aligned} \tag{2.3}$$

The functions $r(t, \sigma)$, $u(t, \sigma)$, $s(t, \sigma)$ comprise a solution of the CCP (2.2).

It can be shown that, since the CCP (2.2) has an analytic solution, the same is true of problem (2.3). A function $\sigma = \sigma_1(t)$ which is a solution of this problem defines in the space of independent variables τ and σ a sonic line $C_1^\mp$ of the solution of the CCP (2.2) issuing from the point $(t = t_*, \sigma = \sigma_*)$. This curve $\sigma = \sigma_1(t)$ must be constructed for $t \leqslant t_*$. Then the function $r = r(t, \sigma_1(t)) \equiv r_1(t)$ defines a sonic line $C_1^\mp$ of the constructed flow in the space of independent variables t and r. $C_1^\pm$ issues from the point

$(t = t_*, r = r_*)$ in the direction of inverse time. The values of the gas parameters are analytic functions on $C_1^{\mp}$

$$\sigma = \sigma_1(t), \quad u = u(t, \sigma_1(t)) \equiv u_1(t), \quad s = s(t, \sigma_1(t)) \equiv s_1(t)$$

determined from the solution of the CCP (2.2).

We emphasize yet again that the CCP is solved in the space of independent variables t and σ and has no singularities. In the space of t and r the solution of problem (2.2) describes a flow which generalizes a simple centred Riemann wave and has a singularity at the point $(t = t_*, r = r_*)$—the limiting values of the gas-dynamic parameters at that point are different on different straight lines through the point.

We now construct the second flow needed to obtain the prescribed discontinuous gas density distribution.

To that end, we consider system (1.1) together with "initial conditions"

$$\sigma(t,r)|_{C_1^{\pm}} = \sigma_1(t), \quad u(t,r)|_{C_1^{\pm}} = u_1(t), \quad s(t,r)|_{C_1^{\pm}} = s_1(t) \tag{2.4}$$

stipulated on the sonic line $C_1^{\mp}$: $r = r_1(t)$, as well as the "boundary condition" (1.3).

Since $\sigma_1(t_*) = \sigma_*(r_*)$, the resulting CCP (1.1), (2.4), (1.3) satisfies the conditions of Theorem 1 and therefore has a unique analytic solution. This solution, first, has the prescribed density distribution at time $t = t_*$ and, second, across the sonic line $C^{\pm}{}_1$ it continuously adjoints the generalized simple centered Riemann wave obtained by solving problem (2.2).

Thus, solving first problem (2.2) and then problem (1.1), (2.4), (1.3), one obtains a solution of the problem of obtaining a prescribed discontinuous density distribution.

If it is required to obtain at time $t = t_*$ a prescribed discontinuous distribution of the gas velocity

$$u(t,r)|_{t=t_*} = u_*(r), \quad u_*(r_*) \neq u_{00}(t_*) \tag{2.5}$$

one can proceed as follows. In the solution of the CCP (2.2) at time $t = t_*$, the gas parameters satisfy a relation

$$u(t,\sigma)|_{t=t_*} = \pm\frac{1}{\delta}s_0^0\sigma + u_0^0$$

where $s_0^0 = \text{const} = s_{00}(t_*)$, $u_0^0 = \text{const} = u_{00}(t_*) \mp s_0^0\sigma(t_*)/\delta$. To solve the CCP (2.2), therefore, we use the value of $u_*(r_*)$ to determine the unique value of σ^* such that $u(t_*, \sigma^*) = u_*(t_*)$. This quantity should then be taken as an initial condition for problem (2.3), solution of which will determine a sonic line $C_1^{\mp}$. After determining $C_1^{\mp}$ and the values of the solution of problem (2.2) on it, we construct a solution of problem (1.1), (2.4), (2.5), which describes the required gas velocity distribution.

When problems of obtaining prescribed discontinuous distributions of the gas parameters are considered in a multi-dimensional setting, one may use previously constructed generalizations of simple centered Riemann waves [10, 11].

In conclusion, we note that a more detailed investigation of meaningful applications of the problem of obtaining prescribed distributions of gas parameters is undoubtedly of interest and deserves a separate consideration.

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